The Atmospheric Mesoscale

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### The mesoscale gap

<table>
<thead>
<tr>
<th>Scale</th>
<th>Synoptic</th>
<th>Mesoscale</th>
<th>Convective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td>1000 km</td>
<td>100 km</td>
<td>10 km</td>
</tr>
<tr>
<td>Time</td>
<td>100 hr</td>
<td>10 hr</td>
<td>1 hr</td>
</tr>
</tbody>
</table>
Convective
10 km
1 hr

Synoptic
1000 km
100 hr

Mesoscale
100 km
10 hr

Convective
10 km
1 hr

(Fiedler and Panofsky, BAMS 1970)
Mesoscale weather phenomena

Mesoscale weather systems

Orlanski (1975)

(Varshney, “Mesoscale Meteorology in Midlatitudes” 2010)
Observed kinetic energy spectrum

Kinetic energy spectrum observed from commercial aircraft (Global Atmospheric Sampling Program; Nastrom et al 1984)

“Universal” shape

$k^{-3}$ power law – QG enstrophy cascade

$k^{-5/3}$ power law – upscale cascade, or downscale cascade, or orography, or gravity wave saturation, or ...?
Large model-based data set

Kinetic energy spectrum at 10 km

COSMO-DE operational analysis
- Nudging to radar and conventional data
- Horizontal resolution: 2.8 km
- Domain: 1200x1300 km, centred over Germany
- 3 years (2014-2016), at 3 hourly intervals

(Selz, Bierdel and Craig JAS 2018, submitted)
Variability of kinetic energy spectrum

Kinetic energy spectrum at 10 km

Relative variability of kinetic energy spectrum

Mesoscale Gap
Correlations with kinetic energy

Correlation of kinetic energy with squared PV anomaly

Correlation of kinetic energy with precipitation

![Graph showing correlation of kinetic energy with squared PV anomaly and precipitation](image-url)
Mesoscale weather phenomena

Timescale:
- 1 month
- 1 day
- 1 hour
- 1 minute
- 1 second

Spatial scale:
- 20 m
- 200 m
- 2 km
- 20 km
- 200 km
- 2000 km
- 10,000 km

Orlanski (1975) scale:
- micro-\gamma scale
- micro-\beta scale
- micro-\alpha scale
- meso-\gamma scale
- meso-\beta scale
- meso-\alpha scale
- macro-\beta scale
- macro-\alpha scale

Mesoscale:
horizontal length scale

(Markowski, "Mesoscale Meteorology in Midlatitudes" 2010)
Mesoscale weather phenomena

- Constant speed $v \sim 10 \text{ ms}^{-1}$
- Weather phenomena concentrated along “advective band”

Mesoscale horizontal length scale

(Orlanski 1975, Markowski, “Mesoscale Meteorology in Midlatitudes” 2010)
Space-time energy spectrum

COSMO-DE simulation
- Horizontal resolution: 2.8 km
- Domain: 7000x4000 km, over Europe and N. Atlantic
- 7 days (from 28 May 2016) output at 2 min. intervals

\( v \sim 10 \text{ ms}^{-1} \)
“advective band”

Inertia-gravity wave dispersion relation for gravest tropospheric mode

(Craig and Selz GRL 2018)
Horizontal and vertical spectra

Effective resolution of model

a) Kinetic energy at 10 km

b) Vertical kinetic energy at 5 km
Horizontal and vertical spectra

Orographic gravity waves

Convective updraughts
Approximations for different regimes

Single-scale asymptotic regimes

Weak Temperature Gradient (WTG) approx.

- Expected for advective band on convective and mesoscale

\[ w \partial_z \theta = Q \]

- Advective motions assumed slower than gravity waves

\[ U \ll NH \]

- Often used for tropics

Validity of WTG measured by:

\[ Fr^2 \sim \left( \frac{U}{NH} \right)^2 \sim \frac{\partial_t \theta + \mathbf{v}_h \cdot \nabla_h \theta}{w \partial_z \theta} \]
Validity of WTG measured by:

$$Fr^2 \sim 0.5$$

$$Fr^2 \sim \left(\frac{U}{NH}\right)^2 \sim \frac{\partial_t \theta + \mathbf{v}_h \cdot \nabla_h \theta}{w \partial_z \theta}$$
Equations for the convective scale

Vertical velocity from heating (WTG approx.):
\[ w^c \partial_z \bar{\theta} = Q^c \]

Horizontal divergence from continuity:
\[ \nabla_c \cdot v_h^c = \frac{1}{\rho} \frac{\partial}{\partial z} (\bar{\rho} w^c) \]

Dynamics from vertical component of vorticity equation:
\[ \partial_{tc} \zeta^c + \nabla_c \cdot (v_h^c \zeta^c) + \nabla_c \cdot (v_h^m \zeta^c) = \nabla_c \cdot (w^c \zeta_h^c) \]
\[ 2D \zeta\text{-conservation advection by } v_h^m \text{ forcing from } w^c \]

**Application**: Balance principle for convective-scale data assimilation

- Divergent wind given to leading order by heating
- Can damp “bad divergence“ (transient gravity waves) without supressing convection
Equations for the mesoscale

Non-divergent at leading order: \( \nabla_m \cdot \mathbf{v}_h^c = 0 \)

WTG at second order: \( \tilde{w}_m \partial_z \tilde{\theta} = Q_m \)

Dynamics from vertical component of vorticity equation:

\[
\partial_{t_m} \zeta^m + \nabla_m \cdot (\mathbf{v}_h^m \zeta^m) + \nabla_m \cdot (\mathbf{v}_h^s \zeta^m) + \nabla_m \cdot (\mathbf{v}_h^c \zeta^c - \mathbf{w}^c \zeta_h^c) = 0
\]

2D \( \zeta \)-conservation advection by \( \mathbf{v}_h^s \) convective source

Preliminary result combining separate two-scale analyses

Application: Forcing of mesoscale by scale interactions

- 2D vorticity conservation couple in vertical by forcing from synoptic and mesoscales
- “Stratified turbulence” with depth scale imposed by forcing terms
Multiscale asymptotics III

Equations for the synoptic scale

Geostrophic wind is non-divergent: \( \nabla_s \cdot \mathbf{v}_h^s = 0 \)

Dynamics from QG potential vorticity equation:

\[
\partial_{t_s} q^c + \nabla_s \cdot (\mathbf{v}_h^s q^s) + \nabla_s \cdot (\mathbf{v}_h^m \zeta^m - \tilde{\mathbf{w}}^m \zeta_h^m) = \frac{f_0}{\overline{\rho}} \partial_{z} \left( \overline{\frac{\rho}{\partial_{z} \bar{\theta}}} Q^s \right)
\]

\( q \)-conservation

mesoscale source
synoptic heating

Application: Mechanism for upscale impact of diabatic heating

• PV source due to forced mesoscale divergent wind – indirect effect of diabatic processes on smaller scales

• Projection of diabatic heating on synoptic scale is only direct effect of heating
1. **Mesoscale gap in variability of kinetic energy spectrum**
2. **Kinetic energy is concentrated along an advective band**
3. **Leading order balance:**
   - Convective scale: Weak temperature gradient
   - Mesoscale: Forced stratified turbulence
   - Synoptic scale: Quasi-geostrophic
4. **Implications**
   - Balance principle for convective-scale data assimilation
   - Mesoscale motions forced by scale interactions
   - Mechanisms for upscale impact of diabatic heating on synoptic scale