Turbulence in mixed-phase clouds

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Introduction

- Mixed-phase clouds: important for radiative balance
  - general circulation models (GCMs) exhibit positive bias in sea-surface temperatures (SSTs) in Southern Ocean
    (SST too warm because too much radiation reaches the surface)
  - possible cause: GCM does not generated sufficient liquid water in mixed-phase cloud to reflect radiation to space
- Turbulence known to be important in generating and maintaining liquid water in mixed-phase clouds (Field *et al.* 2014; Hill *et al.* 2014; Korolev & Field 2008; Korolev & Mazin 2003; Mazin 1986; Heymsfield 1977)
  - not as well studied as warm clouds
- Diffusional growth of droplets and ice particles (no collisional growth)
- Couple Lagrangian stochastic model for vertical velocity with adiabatic cloud model
- Simulation takes place in a layer bounded above and below

Outline of talk

- Introduce model
- Numerical simulations
- Variation with cloud depth and turbulence
- Analytical solutions of simplified models

Adiabatic mixed-phase model

- Assumptions:
  - phase transitions:
    - ice-vapour transition
    - liquid-vapour transition
    - no ice-liquid transition (riming)
  - spherical ice particles
  - liquid droplets and ice particles activated whenever $s_w > 0$
    - droplets and ice particles then have $r = r_c$
    - droplets and ice particles maintained at $r = r_c$ whenever $r < r_c$
    - no contribution towards statistics
  - no curvature or solute effects
  - no mixing with environment
  - no sedimentation or inertia
  - no collisions

Adiabatic mixed-phase model

- Pressure

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\frac{gp}{R_d T} w$$

- Temperature

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{L_v}{c_{pa}} \frac{\mathrm{d}q_l}{\mathrm{d}t} + \frac{L_s}{c_{pa}} \frac{\mathrm{d}q_i}{\mathrm{d}t} - \frac{g}{c_{pa}} w$$

- Supersaturation

$$\frac{1}{1+s_w} \frac{\mathrm{d}s_w}{\mathrm{d}t} = A_1(T)w - A_2(p,T,s_w)\frac{\mathrm{d}q_l}{\mathrm{d}t} - B_2(p,T,s_w)\frac{\mathrm{d}q_i}{\mathrm{d}t}$$

- Droplet radius

$$\frac{\mathrm{d}r_w}{\mathrm{d}t} = A_3(T)\frac{s_w}{r_w}$$

- Ice particle radius

$$\frac{\mathrm{d}r_i}{\mathrm{d}t} = B_3(T)\frac{s_i}{r_i} \quad s_i = \xi(T)(s_w+1) - 1 \quad \xi(T) = e_{sw}(T)/e_{si}(T)$$

- Mixing ratio

$$\frac{\mathrm{d}q_l}{\mathrm{d}t} = \frac{4}{3}\pi \frac{\rho_w}{\rho_a} N_w \frac{\mathrm{d}r_w^3}{\mathrm{d}t} \qquad \frac{\mathrm{d}q_i}{\mathrm{d}t} = \frac{4}{3}\pi \frac{\rho_i}{\rho_a} N_i \frac{\mathrm{d}r_i^3}{\mathrm{d}t}$$

Adiabatic mixed-phase model

- Coefficients A_2 and B_2 depend on s_w because q_v depends on s_w e.g.

$$A_2 = \frac{1}{q_v} + \frac{L_w}{c_p R_v T^2} \quad q_v = (s_w + 1)\frac{e_{sw} R_d}{p R_v}$$

- Often neglected but necessary for conservation of mass
- Temperature dependence (and energy equation) also necessary
  — otherwise supersaturation equation is unphysical
- Constant coefficients in supersaturation equation
  — analogous to Boussinesq-like assumption
- Numerically solve for q_v rather than s_w

Lagrangian stochastic model

- Homogeneous turbulence (Langevin equation)

$$\mathrm{d}w = -\frac{w}{T_L}\mathrm{d}t + \sqrt{C_0\varepsilon}\,\mathrm{d}W, \quad \mathrm{d}z = w\,\mathrm{d}t$$

- Integral time-scale related to dissipation rate and velocity variance by

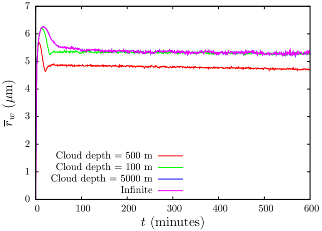
$$T_L = \frac{2\sigma_w^2}{C_0\varepsilon}$$

- Initial velocity drawn from Gaussian distribution: $N(0, \sigma_w^2)$
- Inputs to model:
  - C_0 is constant of proportionality in Lagrangian 2nd order velocity structure function
    $\langle (w(t+\tau) - w(t))^2 \rangle = C_0\varepsilon\tau$
  - fix ε; vary σ_w
- Control relative importance of ballistic vs diffusive behaviour with C_0:
  - motion is primarily ballistic if $C_0 \not\gg 1$
  - motion is primarily diffusive if $C_0 \gg 1$
  - in homogeneous isotropic turbulence $C_0 \approx 6$

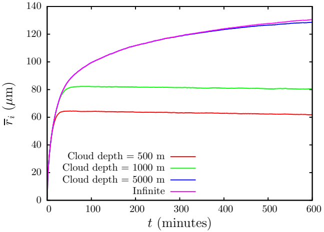
Numerical simulations

- Zero mean velocity
- Finite cloud depth: reflecting boundaries at top and bottom
- Ice number concentration: 10^5 m^{-3}
- Droplet number concentration: 10^8 m^{-3}
- Initial temperature: $T_0 = 267$ K
- Initial supersaturation over water: $s_w = 0$
- Initial droplet radius: $r_w = 0.1$ μm
- Initial ice-particle radius: $r_i = 1$ μm
- Mean kinetic energy dissipation $\varepsilon = 0.01$ m^2 s^{-3}
- Solve using 10^5 – 10^6 Lagrangian particles
- Consider variation of
  - rms velocity: $0.1 \leqslant \sigma_w \leqslant 10$ m s^{-1}
  - cloud depth: $d = 500$ m, $d = 1000$ m, $d = 5000$ m, infinite

Variation with cloud depth



(a) Mean droplet radius

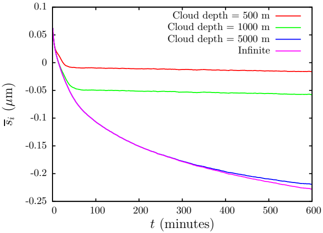


(b) Mean ice-particle radius

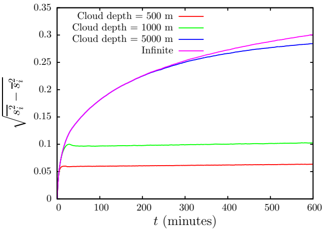
- $\sigma_w = 1$ m s^{-1}
- $\bar{r}_i \gg \bar{r}_w$ since $s_i > s_w$
- Decay rate of ice slower than liquid water

Time scale for sublimation of ice $\gg$ time scale for evaporation of liquid water

Variation with cloud depth



(a) Mean supersaturation

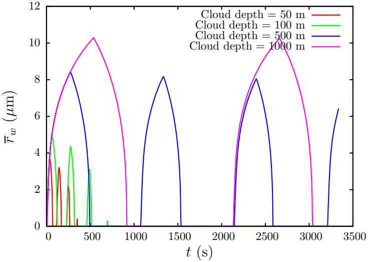


(b) Rms supersaturation

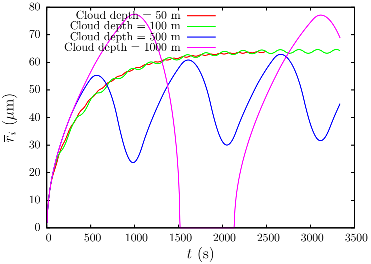
- $\bar{s}_i < 0$ for $t \gtrsim 15$ mins $\Leftarrow$ mainly due to ice
- Turbulent fluctuations can produce (instantaneous) positive values
- Rms supersaturation increases with increasing cloud depth
- Greater depth allows particles to experience larger values of s for longer
- Smaller depth means particles are changing velocity more frequently

Variation with cloud depth

- Analogy with oscillatory flows (Korolev and Field, 2008)
- Constant updraught velocity: 1 m s^{-1}



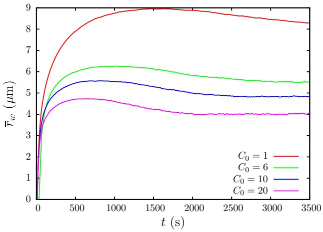
(a) Mean droplet radius



(b) Mean ice-particle radius

Variation with cloud depth

- Variation with C_0: increasing $C_0 \Rightarrow$ more diffusive
  $\Rightarrow$ more rapid change in s but smaller range



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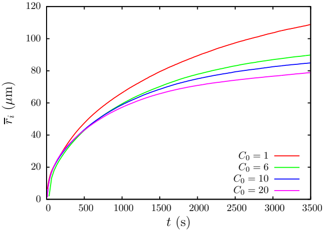
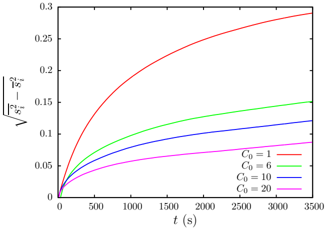


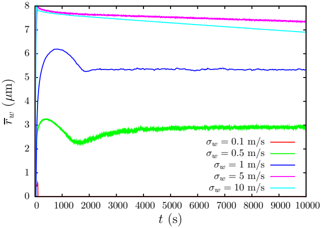
(b) Mean ice-particle radius



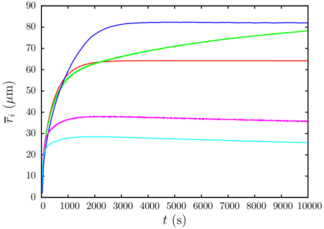
(c) Rms supersaturation

Variation with turbulence

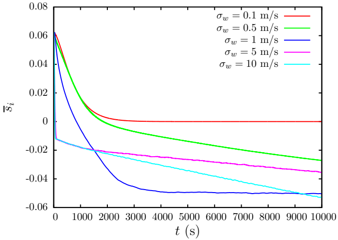
$d = 1000$ m



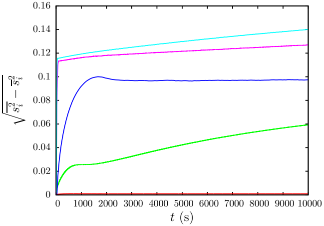
(a) Mean droplet radius



(b) Mean ice-particle radius



(c) Mean supersaturation

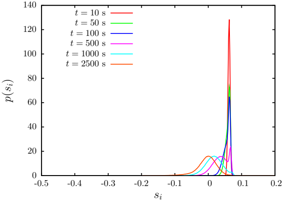


(d) Rms supersaturation

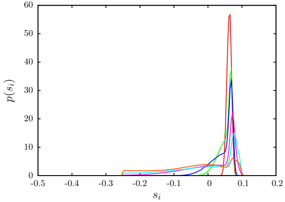
- Liquid water and ice content increase with increasing σ_w
- $\bar{s}_i$ decreases while $(\overline{s_i^2} - \bar{s}_i^2)^{1/2}$ increases with increasing σ_w
- For large σ_w: results affected by cloud depth

Variation with turbulence

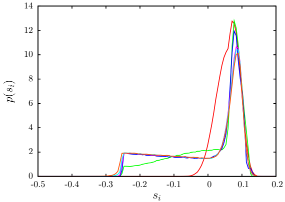
Pdf of s_i



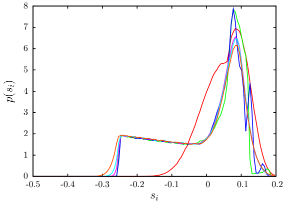
(a) $\sigma_w = 0.5$ m s^{-1}



(b) $\sigma_w = 1$ m s^{-1}



(c) $\sigma_w = 5$ m s^{-1}



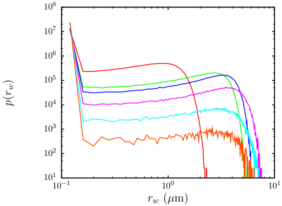
(d) $\sigma_w = 10$ m s^{-1}

- Peak of $p(s_i)$ occurs at liquid water saturation
- As σ_w increases: $p(s_i)$ exhibits increasingly long subsaturated tail
- Range of s_i-values $\lesssim 0$ for which $p(s_i)$ saturates

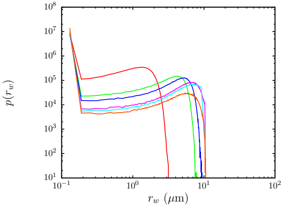
  $\Delta r_w \approx \Delta r_i \approx 0$: Δs_i dominated by w (Gaussian $w \Rightarrow$ uniform s_i)

Variation with turbulence

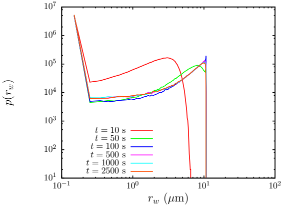
Pdf of r_w



(a) $\sigma_w = 0.5$ m s^{-1}



(b) $\sigma_w = 1$ m s^{-1}



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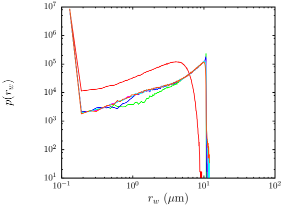
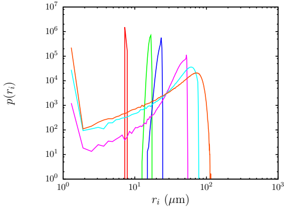


(d) $\sigma_w = 10$ m s^{-1}

- As σ_w increases: $p(r_w)$ saturates for $r_w \lesssim 10\ \mu$m
- Peak forms at $r_w \approx 10\ \mu$m:
    - becomes more pronounced as σ_w increases
    - explains rapid growth of $\overline{r}_w$ at small times

Variation with turbulence

Pdf of r_i



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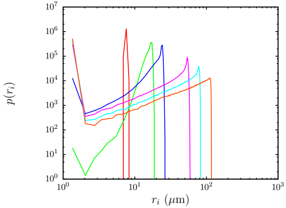
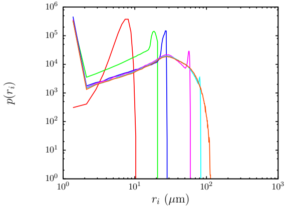


(b) $\sigma_w = 1$ m s^{-1}



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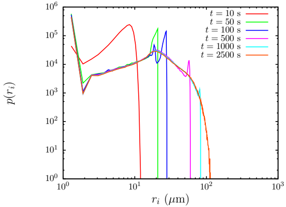
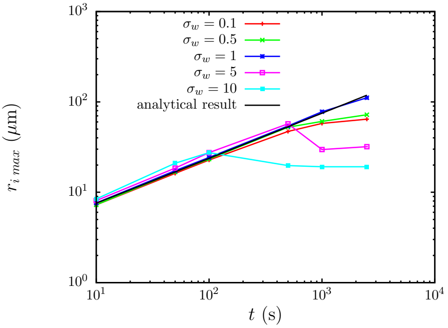


(d) $\sigma_w = 10$ m s^{-1}

- Peak occurs at largest values of r_i
- Peak persists for longer as σ_w increases
- Evolution of this peak governed by peak value of $p(s_i)$
- For these ice particles: $s_i \approx$ constant $\approx$ liquid water saturation

Variation with turbulence

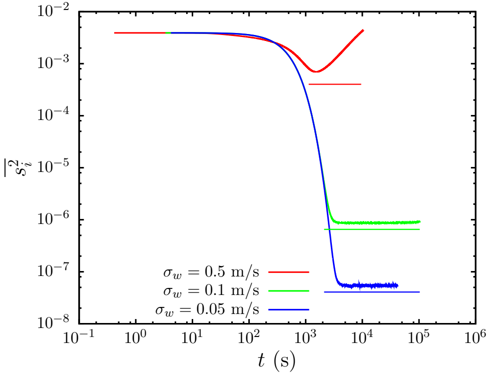


- Analytical result

$$r_i^2 = r_{i0}^2 + 2B_3(s_i|_{s_w=0})t$$

Simplified forms of mixed-phase model

- Constant temperature; constant droplet and ice-particle size
- Appropriate for weak turbulence
- Evaporation of liquid water more rapid than sublimation of ice
  - convenient to remove effect of liquid water
  - re-define supersaturation to include liquid water
  - analytical expression for mean-square supersaturation



Conclusions

- Turbulence is important in generating & maintaining liquid water in mixed-phase clouds
  - can generate liquid water and ice even when $\bar{s}_w, \bar{s}_i < 0$
- Amount of liquid water and ice depends on
  - degree of turbulence
  - cloud depth
- Droplet and ice-particle growth have significant effect on results
- Growth of largest ice particles controlled by mode of $p(s_i)$
- Analytical solutions of supersaturation equation
  - constant coefficients and constant r_w and r_i
- Allow for droplet and ice-particle growth
  - approximate solutions of supersaturation equation are possible
  - several assumptions and approximations

Future work

- Potential subgrid model for GCM
- Extend model to inhomogeneous turbulence
- Model entrainment at boundaries
- Buoyancy effects and temperature fluctuations
- Variable ice-particle number concentration (N_i)
- Fine-scale intermittency

Context: previous studies of turbulent mixed-phase clouds

- Korolev and Field (2008)
  - Random vertical velocity
- Field *et al.* (2014)
  - Diffusion equation for supersaturation
  - Diffusion limit of new model (subject to various caveats)
- Hill *et al.* (2014)
  - Large-eddy simulation (LES)