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[NOTE.—In this memoir temperature as an algebraical symbol is denoted by t, range of temperature on the tercentesimal or centigrade scale by t, and the tercentesimal temperature (° C + 273) by tt.]

For many years, in meteorological practice, it has been customary to represent the result of a sounding with a registering balloon by a graph on a rectangular grid with temperature on a linear scale as abscissa and height on a linear scale as ordinate. The height has been computed from the values of pressure and temperature by the application of Laplace's formula. In the early tables of the soundings pressure was used for that computation only; some time elapsed before it became customary to enter in the tables corresponding values of pressure and temperature. Humidity was given also in the tables, corresponding with successive heights, when the observations were available.

The thermal condition of the atmosphere at any point of the ascent is completely defined by the pressure, temperature, and humidity: the measure of the height is implicitly contained in the knowledge of those elements during the ascent, and as the computation of height is a tedious process, those who are accustomed to work with data for the upper air acquire the habit of using pressure and temperature as the basis of their co-ordinates instead of height and temperature. For these variables a logarithmic scale is more convenient than the linear scale, because it gives a straight line as an adiabatic line for dry air, always parallel to itself for all initial temperatures or pressures.

This method of representation is now in vogue for the Upper Air Supplement of the Daily Weather Report. The height of any point of the trajectory above the starting-point depends upon the initial and final pressures, the intermediate temperatures, and also upon the humidity to a slight extent, but this is unimportant; so that the height is sufficiently indicated by a side scale which is marked on the diagram, day by day, according to the conditions indicated by the graphs.

In like manner the thermodynamical properties of dry and moist air can be set out in diagrams borne on a grid of pressure and temperature each with a logarithmic scale.

It is by bringing the graphs of the actual soundings into relation with the thermodynamic properties of dry and moist air that we can hope to trace in effective detail the physical changes in the earth's atmosphere. For many years my work has been to a certain extent guided by the consideration that the physical processes would be more easily traced if the conditions during the sounding could be set out on a thermodynamic diagram. On such a diagram area should represent "work" in its relation to heat; it should therefore express mechanical energy in dynamical units, and thermal energy too, in the same units.

The co-ordinates which are employed for the ordinary indicator-diagrams of steam-engines, and which are used to illustrate thermodynamic reasoning in Maxwell's *Heat*, are those of pressure and specific volume (volume of unit mass). Referred to those co-ordinates isothermal lines for dry air are rectangular hyperbolas which express the relation $pv = Rt$, and the adiabatic lines for dry air are the curves which express the adiabatic equation for dry air $pv^\gamma = p_0 v_0{}^\gamma$, where γ is the ratio of the specific heat of air at constant pressure to that at constant volume. With diagrams of this type, work is represented by the product of pressure and change of volume or the integral of $p dv$, and the product is evidently expressed in dynamical units—in C.G.S. absolute units if p and v are each expressed in C.G.S. units.

Adiabatic lines for saturated air can likewise be set out on a p, v diagram, but they are not expressed by a simple formula. As a matter of fact, as computed by Hertz and Neuhoff and more recently by J. E. Fjeldstad,[1] they are referred to pressure and temperature, not to pressure and specific volume.

The alternative to the pv diagram is to use temperature t and entropy E as co-ordinates for the representation of the state of the air at any time, measuring temperature as horizontal distance and entropy as vertical. In that case isothermal lines for dry air are vertical lines, and the adiabatic or isentropic lines for dry air are horizontal lines.

The area of a rectangle formed by two pairs of parallel co-ordinates is $(E - E_0)(t_0 - t)$. $(E - E_0)t_0$ is the heat which transforms the entropy from E_0 to E at the temperature t_0, and $(E - E_0)t$ the heat which is returned at the change of entropy back again from E to E_0 at the lower temperature t. If the heat is expressed in dynamical units, with the aid of the dynamical equivalent, we get the energy transformation represented by the rectangular area of the diagram expressed in C.G.S. units if the dynamical equivalent is expressed in C.G.S. units, or, in other words, if the temperature is expressed in degrees absolute, and the entropy is expressed in C.G.S. units per degree of absolute temperature.

Diagrams based upon pressure and specific volume may be regarded as most directly related to the mapping of the atmospheric fields by surfaces of equal pressure and surfaces of equal specific volume, as used by V. Bjerknes and others in their work on *Dynamic Meteorology and Hydrography*; entropy-temperature diagrams are used by Sir A. Ewing in his work on the *Steam-engine*.[2] The relation between the variations of the condition of a substance as expressed in the two methods is the basis of the four thermodynamic relations which are the subject of Chapter IX. of Maxwell's *Heat*.

Failing to get any satisfaction out of an endeavour to express a sounding, or the physical properties of air, on diagrams referred to pressure and specific volume, on account of the awkward shape and dimensions of the graphs, I turned my attention to entropy-temperature diagrams. Comparatively speaking, they are easily constructed for work in the upper air, because for air, or indeed for any gas, the entropy is simply proportional to the logarithm of the potential temperature.

[1] Graphische Methoden zur Ermittelung adiabatischer Zustandsänderungen feuchter Luft. *Oslo, Geofysiske Publikationer*, Vol. III., No. 13, 1925.

[2] See also Callendar's *Properties of Steam*, Arnold, 1920, Appendix II.

In a paper in 1921 on "The Structure of the Atmosphere up to Twenty Kilometres" printed in *The Air and its Ways*, and again in an article on "Thermodynamics of the Atmosphere" in the *Dictionary of Applied Physics*, I gave an account of a method of representing the properties of saturated air by graphs upon a basic rectangular grid with temperature on a linear scale as abscissa and entropy on a linear scale as ordinate; and in 1924, in a lecture before the Mathematical Association at Delft, I explained that the results of a sounding could be set out on the same grid with the thermal curves for saturated air in the groundwork; it was further explained that the energy of dry or saturated air at any point, relative to its environment, was expressed by the area between the graph of the sounding and the adiabatic for dry or for saturated air. Details of the method of working are given in a paper in the *Quarterly Journal*.[3]

The method of representation thus described has been accepted with cordial approval by the International Commission for the Investigation of the Upper Air,[4] and is in practical use in some of the principal meteorological Institutes. The plot of the curve of a sounding on the form prepared for international use with temperature and the logarithm of potential temperature as co-ordinates is called a tephigram.

In 1925, in accordance with the decisions of the International Commission, I had to request that graphs of the soundings made on the international days should be included in the reports contributed for publication in an international volume, and I had also to ask that, for the international air-work represented in the volume, heights should be given in "geopotential" expressed in "dynamic metres," or "geodynamic metres," as we now call them in order to avoid a possible ambiguity. This request contemplates the division of the atmosphere into layers by "level surfaces" which are everywhere normal to the force of gravity. Such surfaces are strictly speaking "horizontal," and are therefore better suited for dynamical work than the geometric height, which is the vertical distance from sea-level expressed in metres or feet.

Making allowance for the variations of gravity with latitude and with height, the geopotential at any point in C.G.S. units is the integral of $g\,dh$, between the point and sea-level beneath it, where g and h are both expressed in C.G.S. units; algebraically the geopotential at any point h is equal to $\int_0^h g\,dh$.

It should be explained that units were chosen by Bjerknes to express geopotential as very nearly equal numerically to the expression of geometric height in metres. This is achieved by taking the gravitational acceleration g in dekametres per second, and the steps of height in metres. Neglecting the variation of gravity, this gives in effect for a height of h metres a geodynamic height $\cdot 981h$, where g is 981 cm./sec.^{2}. The values thus arrived at are now said to be expressed in "geodynamic metres."

The unit which gives this result is somewhat anomalous, as the dimensions of geopotential are L^2/T^2; and if a change from C.G.S. units is made by using the dekametre as unit of length, we get for the geopotential at a height of h metres the value $981h \times 100/10^6$, where g is 981 cm./sec.^{2}. This process may be said to express the geopotential in geodekametres. The unit "geodekametre" is ten times the unit "geodynamic metre."

[3] *London, Q.J.R. Meteor. Soc.*, **51**, 1925, p. 205.

[4] Commission for the Exploration of the Upper Air. Report of the Meeting in London, April 16-22, 1925. M.O. 281, London, 1925.

The Measurement of Height in Soundings by Balloon or Aeroplane.

The request on behalf of the Commission for the expression of height in geopotential has involved the preparation of special forms for the reports of the soundings on the international days, and, incidentally, it raises the large question of the measurement of height from the results of soundings.

On examination, it appears that the habit of regarding g as constant in Laplace's equation implies that we have really been accustomed to work out geodynamic height and to transform the result to an approximate geometric height by using a conventional value for g. It would certainly have been better to keep courageously to geodynamic height until the end of the computation, and transform to geometric height, if that were wished, by the accurate table of equivalents prepared by Bjerknes for his work on *Dynamic Meteorology*.

It is therefore opportune at this stage to examine the relation between the measure of height and the graph of a sounding on the grid of entropy-temperature.

The Relation of Geopotential to Height.

The geopotential at any point is the potential energy, due to gravity, of unit mass at that point. It depends jointly upon the expression of height in feet or metres, and the gravitational acceleration at the station at which the mass, be it balloon, aeroplane, shell, or merely air, is lifted. It is the work which would have to be done upon each unit of mass, pound or gramme or kilogramme, of the balloon, aeroplane, shell, or air, in order to lift it vertically to the specified point, and is therefore a quantity of fundamental importance as a criterion of performance. Dynamically it is a simpler expression of the criterion than the geometric height.

So far as I know, when it is desired to evaluate the work done against gravity in any ascent or sounding, the practice has been to make an estimate of the geometric height by a formula connecting height with the pressure indicated by an aneroid barometer. The formula may be one which is assumed to be of universal application and suitable for every height, like that of Toussaint which has been proposed for the graduation of altimeters, or it may be adjusted for successive stages of the records of pressure and temperature as described in W. H. Dines's account of the method of evaluating the records of a meteorograph.

Evaluation of Geodynamic Height from a Graph of Pressure in Relation to Temperature.

An evaluation of the height can be obtained graphically from a measurement of area on the graph of the sounding on a grid of pressure on a logarithmic scale and of temperature on a linear scale.

On semi-logarithmic paper, designed by W. H. Dines for the evaluation of heights, with 1 metre on the log scale to the difference between log 1000 and log 100, and 0·5 cm. to a degree on the linear scale (which on the printed form is assigned to height):

Geopotential in C.G.S. units

$$= 6 \cdot 6 \times 10^6 \times \frac{1}{50} \text{ (area in cm.}^2 \text{ between the curve and line of absolute zero)} \quad . \quad . \quad (1)$$

If the planimeter, or other device for measuring area, is only used up to some selected line, say 200 tt, the necessary addition can be computed in square centimetres as 100 × length of the pressure range in centimetres.

Taking, for example, an ascent with an equivalent mean temperature of 250 tt from 1000 mb. to 100 mb., the area measured is 25 × 100 cm.^{2}, the additional area computed, 100 × 100 cm.^{2}, and the total area enclosed 12,500 cm.^{2}.

The geopotential in C.G.S. units

$$= 6 \cdot 6 \times 10^6 \times \frac{12500}{50}$$

$$= 1650 \times 10^6 \text{ C.G.S.}$$

Changing the unit of length from a centimetre to a dekametre (1:1000), the dimensions being L^2/T^2, we get the geopotential equal to 1650 geodekametres, or 16,500 geodynamic metres, or, by the table, 16,862 metres.

The evaluation of the formula starts from the ordinary Laplace equation $-dp = g\rho dh$. The integral of gdh is the geopotential which we are seeking, and $\rho = p/(Rt)$.

In C.G.S. units R is $2 \cdot 870 \times 10^6$, the modulus for changes of logarithmic scale is 2·30, hence

$$gdh = -Rtd(\log_e p),$$

$$= -2 \cdot 870 \times 2 \cdot 30 \times 10^6 \; td(\log_{10} p)\,;$$

t as a length in centimetres is equal to half the number of degrees, and $\log p$ as a length in centimetres is 100 times the logarithm (because the length of a difference 1 is 1 metre). Hence, if $td(\log_{10} p)$ be counted as area in square centimetres, the numerical value of the integral $\int gdh$ will be $(2 \cdot 870 \times 2 \cdot 30 \times 10^6 \times 2)$ (area in sq. cm.)/100, which corresponds with equation (1).

The Evaluation of Height from a Tephigram.

I propose to show that a tephigram representing a sounding can be used in a simple and effective manner for giving the geodynamic height of any point on the curve provided that the curve is drawn with sufficient accuracy and distinctness to permit the use of a planimeter, or some other method of measuring the area of the strip which extends from the curve (or a selected portion of it) along the lines of equal entropy to the boundary of the form.

The demonstration will be illustrated by three figures reproduced from forms specially prepared for tephigrams. The forms carry, in blue, the basic grid of temperature and megatemperature, *i.e.* potential temperature with 1000 mb. as standard pressure; and superposed in brown are certain properties of saturated air in relation to temperature and megatemperature, namely (1) lines of equal pressure, (2) lines of equal vapour-content for the saturation of a kilogramme of dry air, and (3) adiabatic lines for saturated air. These lines appear in the reproductions though they are not required in the demonstration, except in one case, and are therefore not provided with numbers. The saturation adiabatics are the full lines which start nearly at right angles to the line of 1000 mb., and curve away to the right; the pressure lines drawn for 1000 mb., 900 mb., etc., to 100 mb., are chain lines, nearly straight and nearly at 45° to the axes; and the vapour-content lines are dotted lines, also nearly straight, and not very different from the vertical lines of equal temperature; beginning from the left they show 20 g., 16 g., 12 g., 8 g., 4 g., 2 g., 1 g., 0·5 g., and 0·1 g., respectively.

The thermodynamic basis of the method of evaluating the geopotential from a tephigram is as follows:

The equation connecting pressure with height is

$$\mathrm{d}p = -g\rho\,\mathrm{d}h,$$

and $g\mathrm{d}h$ represents the increase of geopotential for the step $\mathrm{d}h$. Hence if Γ represent the geopotential at any point

$$\mathrm{d}p = -\rho\,\mathrm{d}\Gamma.$$

Using the ordinary gas-equation $p = R\rho t$, we get $\mathrm{d}\Gamma = -Rt\,\mathrm{d}(\log p)$.

The pressure p is necessarily involved with the standard pressure p_0 in the values of temperature t and potential temperature T by the adiabatic equation

$$\log p_0 - \log p = \frac{\gamma}{\gamma - 1}(\log T - \log t).$$

Treating p_0 the standard pressure as constant, and T as variable in the track of the sounding, we get by differentiation

$$-\mathrm{d}(\log p) = \frac{\gamma}{\gamma - 1}\{\mathrm{d}(\log T) - \mathrm{d}(\log t)\}.$$

Hence

$$\mathrm{d}\Gamma = R\frac{\gamma}{\gamma - 1}t\,\mathrm{d}(\log T) - R\frac{\gamma}{\gamma - 1}\mathrm{d}t.$$

But, as we have said, the entropy of unit mass of air is simply proportional to the logarithm of its potential temperature; the factor of proportionality is the specific heat at constant pressure c_p; hence, when work and heat are both expressed in dynamical units, d log T is $\mathrm{d}E/c_p$, where E represents entropy. The equation becomes

$$\mathrm{d}\Gamma = c_p t\,\mathrm{d}(\log T) - c_p\mathrm{d}t.$$

Integrating,

$$\Gamma = \int t\,\mathrm{d}E + c_p(t_0 - t), \quad . \quad . \quad . \quad . \quad (2)$$

t_0 being the initial temperature.

On the temperature-entropy diagram represented in Fig. 1, the first term expresses the energy in dynamical measure represented by the area bounded by the tephigram AP on the left, and the line of zero temperature on the right, and between the two horizontal lines of equal entropy drawn as AD and PC from the initial point A and the point P of the tephigram. The line of zero temperature is not shown in the diagram; it is parallel to CD and 200 mm. on the right of it since the graduation of temperature is one degree per mm. From the thermal point of view the area would represent in dynamical units the amount of heat communicated from the environment to the rising mass during its ascent in order to bring it into equilibrium with the environment at each stage.

The area between the tephigram and any selected vertical CD can be measured with a planimeter, and the remainder computed as the product $(E - E_0) \times 200$. It has an advantage over any other method of computing Γ, in that it takes account of all the sinuosities which are indicated in the curve of ascent.

The second term $c_p(t_0 - t)$ is easily calculated. Thermodynamically it is the energy in dynamic units which is taken from the material of the rising mass on account of the change of its temperature. The factor c_p in dynamical units is the dynamical equivalent of the specific heat of air at constant pressure, which in C.G.S. units is $4 \cdot 2 \times 10^7 \times \cdot 23$, or $9 \cdot 66 \times 10^6$.

The two terms taken together give in dynamical measure the whole amount of heat involved in the process, viz.: first, the part communicated from outside if there is no source within; and secondly, the part contributed from the material itself.

By writing of the heat taken from the environment to bring the rising

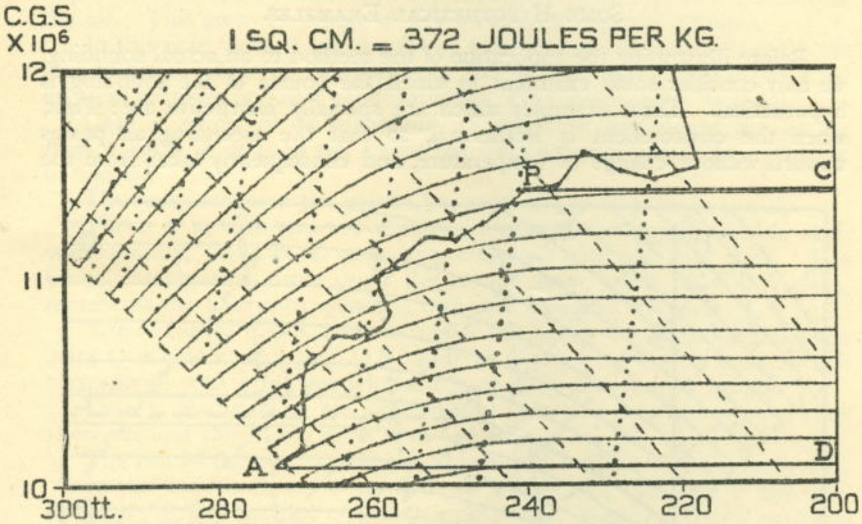

FIG. 1.—Tephigram of a sounding at Trappes on January 18, 1923.
Temperature is represented on a horizontal scale of 1 millimetre to 1 degree centigrade, and entropy on a vertical scale as marked on the left-hand side.
One square centimetre represents $3 \cdot 72 \times 10^6$ ergs per gramme.

mass into thermal equilibrium with the environment at each stage of the process, we have implied that the ascent is "forced" by the application from outside of whatever energy may be necessary; but the process of computation is in effect independent of whether the ascent is automatic or forced. In the cases of automatic ascent which are contemplated in meteorological practice, the energy necessary for the ascent has to be supplied beforehand in the case of dry air, or furnished by automatic condensation of water-vapour during the ascent in the case of saturated air. In the case of forced ascent the heat necessary to bring the ascending air into equilibrium with the environment at each stage may be regarded as supplied by conduction from the environment, which, itself, may be considered as infinite in extent, and therefore unaffected as regards its own temperature, just as the temperature of a vessel of water may be regarded as unaffected by the thermometer used to measure it.

The relation between height and geopotential is of course a matter of direct calculation from the value of gravity, and it may be claimed that the evaluation of the geopotential by the method here indicated is a more satisfactory way of arriving at a determination of height than any of those which are usually employed to interpret the change of pressure in terms of height.

It will be remembered that the tephigram is obtained from the original graphs of pressure and temperature in relation to time, when the meteorograph carries a clock, or from the graph of pressure in relation to temperature on the meteorographs which have no clock. No assumption of any kind is made in regard to the relation of pressure and height. The transformation from pressure to potential temperature for an agreed standard pressure can be computed numerically for a succession of points on the curve, or made graphically in the manner described in the *Quarterly Journal*, 1925.

Some Hypothetical Examples.

Before illustrating the application of the method to an actual sounding, we may consider some examples in which the course of the sounding is hypothetical. Three examples which are specially instructive are: First, when the environment is isothermal, so that the ascending air passes upward without change of temperature, and consequently takes from the

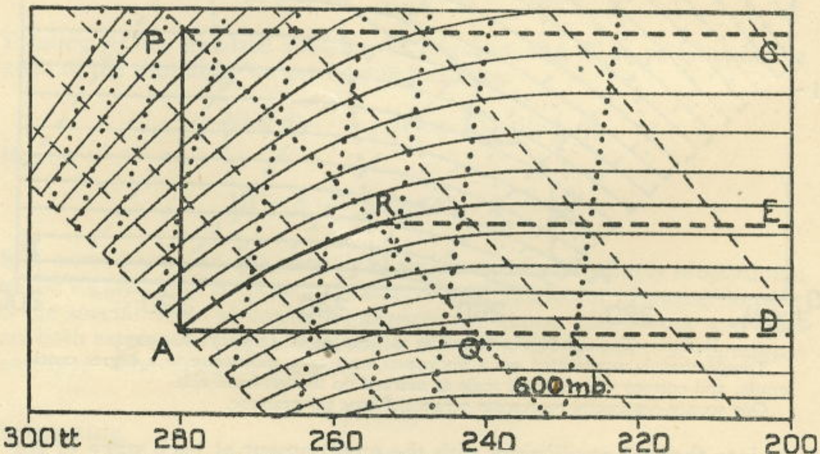

Fig. 2.—Some hypothetical examples of the ascent of air from a point A at 280 tt and 1000 mb. to a point at 600 mb. : (1) in an isothermal atmosphere AP, (2) in an atmosphere in convective equilibrium AQ, (3) along a saturation adiabatic AR.

environment enough heat to compensate completely the adiabatic cooling due to change of pressure. Secondly, when the environment is in a state of convective equilibrium for dry air. In that case there is no addition to the entropy of the ascending air during the ascent, and therefore no extraneous heat is required. Thirdly, we take the case of ascent along the adiabatic for saturated air which passes through the starting-point. In this case the ascending air takes up the heat of evaporation which is set free on condensation, and requires no more. At the same time it loses temperature, as shown by the change of position of the point in the diagram.

In order to exhibit the relationship between the three cases, we have taken all three ascents from a common starting-point A (Fig. 2), and each of the three lines of ascent, AP, AQ, AR, terminates where it crosses the line of pressure marked on the diagram as 600 mb. The computation in the several cases is as follows:

Case 1. *Ascent in an isothermal atmosphere at* 280 *tt from the point A at* 1000 *mb. to the point P at* 600 *mb.*

Draw the lines of equal entropy PC and AD to meet the line of temperature for 200 tt in C and D.

Then the first term of equation (2) is given by the dynamical equivalent of the area APCD, and the area between CD and the line of zero temperature. In C.G.S. units the area must express energy in C.G.S. units per gramme.

The first area is 8 cm. × 4 cm., and the second 20 cm. × 4 cm. The whole area is 112 square centimetres. The diagram is drawn to such a scale that one square centimetre represents 372 joules per kilogramme of dry air. This gives an energy of $372 \times 112 \times 10^4$ ergs per gramme.

In this case there is no change of temperature, and, consequently, no second term. The full expression of the geopotential at P is $4{\cdot}165 \times 10^8$ ergs per gramme or 416·5 geodekametres, 4165 geodynamic metres.

Case 2. *Ascent in an atmosphere in convective equilibrium for dry air from a point A at* 1000 *mb. where the temperature is* 280 *tt to a point Q at* 600 *mb.*

The temperature at the point Q as read on the diagram is 241 tt.

There is in this case no change in entropy during the ascent, and, consequently, there is no area to correspond with the first term. The whole computation turns upon the second term, which, in C.G.S. units, represents the heat corresponding with the change of temperature of 39 t.

The factor for C.G.S. units is $9{\cdot}66 \times 10^6$, hence the geopotential at the point Q is $9{\cdot}66 \times 10^6 \times 39$ C.G.S. units or $3{\cdot}765 \times 10^8$ C.G.S. units, which corresponds with 376·5 geodekametres, or 3765 geodynamic metres.

Case 3. *Ascent along the adiabatic for saturated air from a point A at* 1000 *mb. and* 280 *tt to a point R at* 600 *mb.*

The run of the adiabatic line is shown in the diagram by the curve AR.

In this case both terms have to be evaluated. The first is the area between AR and the line of zero temperature. We have first to evaluate, by planimeter or otherwise, the area ARED, and then add the area between ED and the zero line of temperature.

The area ARED by estimation of squares works out at 10·2 square centimetres, and the remainder at 30 square centimetres; the total is 40·2 cm.^{2}, which corresponds with 14,950 joules or $1{\cdot}4950 \times 10^{11}$ ergs per kilogramme, or $1{\cdot}4950 \times 10^8$ ergs per gramme. The second term is the equivalent of a change of temperature of 26 t as read on the diagram, which gives $26 \times 9{\cdot}66 \times 10^6$ C.G.S., or $2{\cdot}510 \times 10^8$ ergs per gramme. The two terms together give $4{\cdot}005 \times 10^8$ ergs per gramme, or 400·5 geodekametres, 4005 geodynamic metres.

Thus the geopotentials at the three points P, Q, R, on the same pressure-surface are 416·5 geodekametres for an isothermal atmosphere, 400·5 for the saturation adiabatic, and 376·5 for the atmosphere in convective equilibrium. The equivalents in geodynamic metres are 4165, 4005, and 3765, or at latitude 52°, 4248, 4085, 3840 metres respectively.

The order of the heights at which the same pressure is reached in the three examples is in accord with the general principle that, in an isothermal atmosphere, pressure falls off more slowly than in an isentropic atmosphere in which the lapse-rate of temperature is the maximum possible for equilibrium. In the intermediate case in which the lapse-rate is approximately one-half of the adiabatic rate, the height reached when the fixed pressure is attained will be intermediate between the isothermal case and the isentropic case, as the computed figures indicate.

The Method in Practice.

As an example, the curve for Sealand of September 14, 1926, is selected. The original diagram (Fig. 3) is plotted on the usual form, in which 1 sq. cm. represents 372 joules per kilogramme.

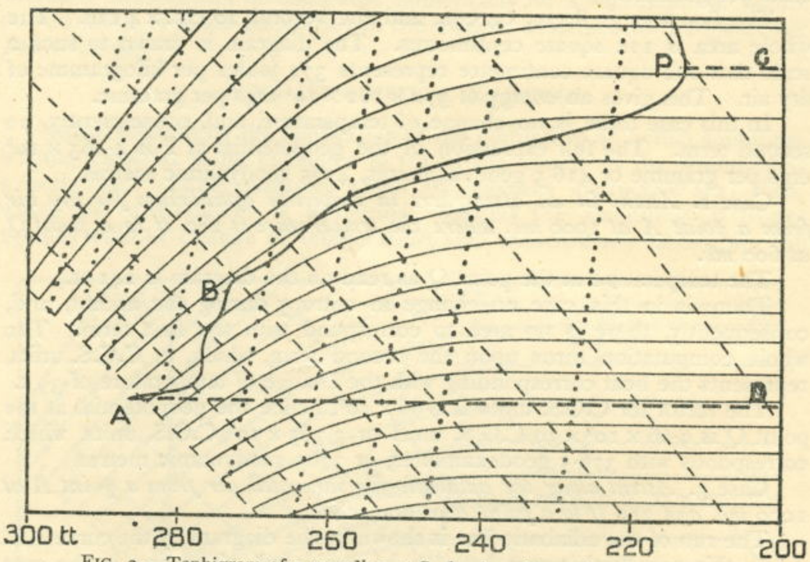

FIG. 3.—Tephigram of a sounding at Sealand on September 14, 1926.
Temperature is represented on a horizontal scale of 1 millimetre to 1 degree centigrade, and entropy on a vertical scale. The vertical side of the rectangle of the diagram represents $2\cdot5 \times 10^6$ ergs per gramme per degree.
One square centimetre represents $3\cdot72 \times 10^6$ ergs per gramme.

The curve was traced on to graph paper in millimetres, and the geodynamic height of the point P over that of A found.

The area between the curve and the 200 tt line works out, by counting of millimetre squares, at $25\cdot07$ cm.^{2}, the area between the 200 tt line and the absolute zero line is 90 cm.^{2}. Thus the total area between the curve and the line of absolute zero is $115\cdot07$ cm.^{2}, or 429×10^6 ergs per gramme.

Bearing in mind that the energy is to be expressed in dynamical units, the additional term $c_p(t_0 - t)$ is equal to $9\cdot66 \times 10^6 \times 73\cdot5$, or 711×10^6 ergs per gramme.

Hence, by adding the two terms, we obtain $\Gamma = 1140 \times 10^6$ ergs per gramme, or 1140 geodekametres, or 11,400 geodynamic metres, which is converted by table to 11,746 metres.

In the same way the geodynamic height of B, where tt is 274, above A, works out at $287\cdot7$ geodekametres, 2877 geodynamic metres, and, by the table, 2920 metres. So in like manner the geopotential and height of any point on the curve can be calculated.

It is not improbable that the use of temperature-entropy diagrams or temperature log (pressure) diagrams in this manner is well known to the few meteorologists who are accustomed to deal with the atmosphere from the thermodynamic point of view, but I am unable to give any reference in support of that probability, and other meteorologists interested in the study of the upper air may be in like case. I am taking the opportunity of referring to the subject because it seems probable that the relative importance of the two terms in different ascents, or in different parts of the same ascent, may afford a useful means of classifying the different conditions of the atmosphere which are disclosed in the study of the upper air. Moreover, the time seems to be opportune for using the idea of geopotential in the evaluation of the performance of balloons or aeroplanes much more explicitly than has hitherto been customary.

Mr. L. H. G. Dines has been good enough to examine the practical working of the method, and has suggested a technique which I hope he may find an opportunity to communicate. Mr. J. S. Dines has also had the method under his notice, and has practised it with success.

ABSTRACT.

Geopotential as the integral $\int g\,dh$ can be evaluated graphically from a curve representing the relation of temperature, on a linear scale, and pressure, on a logarithmic scale, during the sounding. The geopotential can be converted into geometric or geographic height by the tables in *Dynamic Meteorology and Hydrography*, by V. Bjerknes and others.

Being based upon the relation of pressure to temperature during the sounding, the curve of a tephigram (a representation of the sounding on a form with temperature and log (potential temperature) as co-ordinates) also lends itself quite easily to the determination of the geopotential at any point of the curve, and thence of the height.

The numerical value in C.G.S. units of the difference of geopotential between two points A and P is made up of two parts.

The first part represents the supply of heat necessary to keep the unit mass of ascending air in thermal equilibrium with its environment at each stage of the sounding. It is represented algebraically by $\int t\,dE$; and, on the diagram of which t and E are co-ordinates, by the area of the figure bounded by the curve AP of the tephigram on the left, the line of zero temperature on the right, and the two lines drawn horizontally through A and P respectively to the line of zero temperature.

The second part is the dynamical equivalent in C.G.S. units of the heat which corresponds with the lapse of temperature of unit mass of air between A and P.

The two parts in dynamical measure added together give the equivalent of the total amount of energy required to effect the change in the geopotential between A and P. Both parts are easily evaluated with the aid of the tephigram.

The following cases are considered:

1. An isothermal atmosphere for which the line AP is vertical. There is no lapse of temperature; the second part is zero; only the first part counts.

2. An atmosphere in convective equilibrium for dry air, for which the line AP is horizontal. There is no change of entropy, there is no area to be measured; the first part is zero, and only the second part counts.
3. Ordinary conditions under which both temperature and entropy change and both parts count. In the small experience hitherto available the second part is the more important.

An algebraical demonstration is given of the formula upon which these statements are based, and the three cases are illustrated by diagrams, two of which deal with actual soundings.

Note added 13th May 1927.

ERRATUM.—On p. 113 the numerical value in dynamical units assigned to c_p is $\cdot 23 \times 4\cdot 2 \times 10^7$ or $9\cdot 66 \times 10^6$. From the values given in the *Computer's Handbook* for the specific heat of dry air and the dynamical equivalent of heat the figures should be $\cdot 2417 \times 4\cdot 18 \times 10^7$ or $1\cdot 010 \times 10^7$ C.G.S. units, and this value should accordingly be used instead of $9\cdot 66 \times 10^6$ in pp. 113, 115, 116.

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