Isentropic relative-flow analysis and the parcel theory

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SUMMARY

A discussion is given of the justification and usefulness of analysing on isentropic charts observations of the flow relative to a mature large-scale motion system under the assumption that the system is in a steady state. It appears that trade wind air may rise over a front into a jet stream and that by an extension of the parcel theory into three dimensions the speed of a jet stream may be related to an area on the aerological diagram.

1. THE ANALYSIS PROBLEM

Atmospheric observations are interpreted in terms of motion systems having characteristic patterns of the distribution of the meteorological elements and characteristic time and space scales. Each system has an evolution between times when it is no longer distinguishable from other systems. The number of individual measurements needed to represent even a single system amounts to several powers of ten and is obtained virtually only for systems of very large scale whose life is at least a few days and which are therefore important in day-to-day forecasting. Their study is made more difficult by the influence of smaller-scale systems whose presence and intensity are inadequately revealed by the available observations, and additional complexity arises from the interaction not only with other systems but also with the complicated features of the earth's surface.

Consequently the *analysis* of the observations aims to reduce the vast amounts of data into models, which are to be as simple and comprehensible as possible without omitting the essential physical processes determining the behaviour of the systems. The models are at first graphical, but their ultimate expression is mathematical. In this note we outline an unconventional analysis technique whose value is being explored.

2. ISENTROPIC ANALYSIS

It is hardly possible to visualize the four-dimensional structure of the motion systems except by reference to patterns drawn on paper, representing sets of two-dimensional sections (in which one coordinate is variously horizontal or vertical dimension, or time). Until recently only the conditions near the Earth's surface could be described satisfactorily, but even before the extensive development of the network of radiosonde stations, Rossby and his collaborators (1937) prepared maps including the flow aloft in (isentropic) surfaces of constant potential temperature. The stimulus of aviation, which provided the expansion of the observing network, probably also influenced the form of analysis, and it has become conventional to divide the atmosphere into isobaric sections (most of the time aircraft fly in isobaric surfaces).

Isentropic maps have an advantage over isobaric maps in that they contain a direct indication of vertical motion and of the physical processes at work. In so far as the air motion is adiabatic it is in the surface described in the isentropic map; in general, of course, this condition is not fulfilled, but adiabatic motion has to be used as a reference process, and it may be turned to advantage that while in some regions the motion is nearly adiabatic and therefore practically in the plane of the isentropic map, in others the departure is quite evident, so that the location and even magnitude of pronounced non-adiabatic processes are indicated.

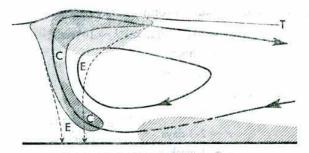


Figure 1. Schematic representation of flow in deep tropospheric convection (either cumulonimbus or large-scale slope convection), projected upon a vertical section. The hatched area at the surface represents the layer of small-scale convection, within which subsiding air has its potential temperature raised. Air from this layer enters a cumulonimbus or large-scale circulation and ascends into the upper troposphere. Condensation, C, of water vapour occurs, and small-scale convection may develop again in the cloud system where there is cross-hatching. Precipitation falls from the cloud (pecked lines) and there is some evaporation, E, outside it. The descending branch of the large-scale circulation is left open on the right to indicate that it continues in several motion systems, accompanied by a slow radiative loss of potential temperature. T marks the tropopause.

The deep tropospheric motion system (associated with cyclones or with thunderstorms) can be represented schematically projected upon a vertical section as in Fig. 1. In this diagram the pattern of vertical motion is markedly asymmetrical, the upward velocities being large and concentrated into a small part of the system where the condensation of water vapour occurs. In and near this region the flow is rapid and may be nearly adiabatic (i.e., the changes of state are nearly those in the appropriate dry or moist reference process), and a large part of the condensed water is precipitated. Outside it, descending motion is cloud-free and, for return to the surface layer in the same latitude, is limited to the slow rate determined by radiative heat loss. The magnitude of this loss is set ultimately by the rate of absorption of solar radiation in the troposphere and at the ground, and on the average is equivalent to a tropospheric cooling rate of between 1° and 2°C/day. Considering that the difference between the actual temperature (say 20°C) and equivalent potential temperature of warm air near the ground may be up to 40°C, this implies the rather long descent-period of up to 20 days or more.

The mature large-scale motion systems are observed to occupy typically about one-fifth of the middle-latitude belt in which they occur, so that air in both the upper and the lower troposphere flows through them at a typical relative speed of 15 m sec⁻¹ or more within 2 or 3 days. In the diagram, therefore, the arms of the descending circulation have been left open to indicate that the descent continues in other systems, and that there is no definite boundary on that side. Air approaching the surface enters a layer of variable but generally rather small depth in which energy (sensible and latent heat) is introduced (by a small-scale convection involving thorough mixing) at about the rate quoted above, so that after only a few days the air is again ready for ascent. Within this limited region of small-scale convection trajectories have only a statistical meaning, and there is the greatest departure from adiabatic flow, but outside it the departure is barely significant over a period of 2 or 3 days. Accordingly, over most of one system the motion can be regarded as approximately adiabatic, sufficiently so to justify its representation on isentropic charts.

3. RELATIVE-FLOW ISENTROPIC ANALYSIS AND THE STEADY-STATE ASSUMPTION

In the examination of the physical structure of a mature system it can be further supposed that the flow is in a steady state during a period τ short compared with its recognizable life T, but rather long compared with the time t taken for air to traverse the

region of condensation. For the thunderstorm we may have :

 $t \sim 10 \text{ km/}10 \text{ m sec}^{-1} \sim 10^3 \text{ sec}^{-1}$ $\tau \sim 1 \text{ hour}$ T ~ few hours,

while for the large-scale convection

 $t \sim 1 \, \text{day}$ $\tau \sim 2-3 \text{ days}$ $T \sim 1$ week

Combining this supposition with the assumption of quasi-isentropic motion over periods of up to 2-3 days, we see that on a surface of constant potential temperature the streamlines of the wind relative to a particular system are trajectories. They are not the trajectories relative to the ground, although these may fairly readily be derived from them. But in so far as the system is in a steady state we may seek to relate the successive states observed along the trajectories by physical conservation laws, since these states would be

experienced by particular samples of air moving through the system.

Apart from the doubt about the validity of the steady-state assumption, a number of difficulties arise. Firstly, the system and its motion have to be defined. The most definite part of a system is the edge of the condensation region recognized as the squall front in the thunderstorm and the major cold front in the large-scale motion system. In exploring the usefulness of representing the large-scale systems by the relative flow on isentropic surfaces, we have so far taken the velocity of the system under study to be the average zonal motion of the front or of the axis of the cold trough, but some other convention may prove more profitable. Secondly, since much the greater part of the volume occupied by a large-scale system is unsaturated, it is reasonable to represent the flow in surfaces of constant dry-bulb potential temperature. Within the condensation region, however, the trajectories may be more approximately in surfaces of constant wet-bulb potential temperature. The consequences have somehow to be taken into account and displayed.

Other difficulties arise where the trajectories pass through the region of small-scale convection, within which the motion is essentially not adiabatic, and the trajectories have only a statistical sense. Where the small-scale convection occurs diurnally over areas of such limited geographical extent (e.g., Spain) that air can flow across them in less than one

day, a confusing unsteadiness is introduced.

Nevertheless, the technique of representing the motion-systems by the relative-flow isentropic analysis has some interesting advantages. For example, observations made at various times within the period τ can be displaced appropriately in space and entered upon the same chart. In this way they are much more readily incorporated into the analysis than when a sequence of charts at intervals of time is examined for consistency of evolution.

Their most appealing immediate virtue is in providing a clearer picture of the convection and the field of vertical motion than the conventional isobaric analyses. Apart from its use in analyses of the large-scale situations associated with severe local storms, to be described elsewhere, the technique has been explored in more detail using data from western Europe on 12 July 1961 (when a severe thunderstorm occurred near Brussels). We have experimented with charts representing the average flow between two narrowly separated (e.g., by 2°C or 4°C) isentropic surfaces, using the plotted value of the pressure separation as an index of lapse-rate, and (outside regions of small-scale convection) using the implied constant mass-flux between adjacent streamlines as a constraint upon their construction. We have also examined how closely water vapour mixing-ratio and potential vorticity are conserved along the streamlines, with encouraging results, in spite of deficiencies in the data (for example, inaccurate humidity measurements in the middle and high troposphere, and insufficient observations to reveal strong horizontal wind shears and concentrations of vorticity).

4. Conservation of energy along trajectories: the 'extended' parcel theory

With the assumption of the steady state and, additionally, that friction and other external forces can be neglected and that the motion is adiabatic, the equation of conservation of energy along the streamlines on the relative-flow isentropic charts becomes (Bjerknes 1917; see also, e.g., Haurwitz 1940, p. 240),

$$v^2/2 + gz + c_v T + p/\rho + Lx = \text{const.},$$
 (1)

where

v is relative speed

z is height

cv is the specific heat of air at constant volume,

p is air pressure,

ρ is air density,

L is the latent heat of condensation, and

x is the mixing-ratio of condensed water,

or

$$v^2/2 + gz + c_p T + Lx = \text{const.}$$
 (2)

where c_p is the specific heat of air at constant pressure.

The identical sequence of changes of state (as measured by T, p and x) occur during vertical adiabatic displacement in a horizontally uniform hydrostatic atmosphere with the appropriate adiabatic lapse-rate and are there by definition associated with height z' above the same reference level according to the equation

$$gz' + c_p T + Lx = \text{const.}$$

Along the relative-flow isentropic streamlines we have, by subtraction,

$$v^2/2 + g(z - z') = \text{const.},$$

and in particular,

$$v_1^2 - v_0^2 = 2g (\Delta z' - \Delta z),$$
 (3)

where v_0 is an initial (relative) speed, v_1 a speed attained after an ascent Δz , and $\Delta z'$ is the height change in the adiabatic atmosphere corresponding to the same change of state. We call this a result of an 'extended' parcel theory (a somewhat similar approach was made by Danielson (1961) to the problem of trajectory construction).

5. Application to cumulonimbus

The application of this expression to the cumulonimbus has already been given (Ludlam 1963; p. 22). In this case the term on the right is usually almost wholly the work done by buoyancy, as given by the ordinary parcel theory and represented by an area on an aerological diagram between an adiabatic line and the curve representing the state of 'the environment.' The ordinary parcel theory, however, neglects the term in v_0 . Since those clouds for which the steady-state and adiabatic assumptions may be reasonable are observed to move relative to the low-level wind at speeds of up to about 20 m sec⁻¹, the contribution of this term may be significant. For example, the updraught can be imagined to attain this speed, sufficient for the production of an intense thunderstorm or hailstorm, with negligible buoyancy, or an inconsiderable 'positive area' on the aerological diagram.

6. Application to large-scale convection

In the large-scale convection the strongest accelerations similarly appear along trajectories which pass through the condensation region. The model for the large-scale motion system (Fig. 2) is very similar to the model for the well-organized cumulonimbus,

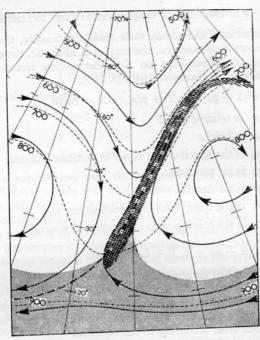


Figure 2. Schematic representation of relative flow in a mature summer large-scale slope convection (trough-ridge) system over an ocean, in a surface of constant (dry-bulb) potential temperature (θ about 30°C). The height of the surface is shown by thin pecked lines labelled in mb. The major (cold) front is shown as a dot-dash line in the confluence between two principal air streams. The stippled area in the south shows air which has been modified by small-scale convection. The hatched area is a cloud system formed where some of this air rises above the isentropic surface, producing low clouds liable to small-scale convection (cross-hatching) in the south, subsequently middle-clouds, and finally cirrus clouds near and to the right of the axis of the jet-stream of the high troposphere over the front in middle latitudes. The cirrus cover diminishes where the flow turns north-westerly.

except that the horizontal-scale is very much greater (Ludlam 1963; p. 16). In the formulation of the ordinary parcel theory, involving only vertical motion through uniform and undisturbed surroundings, the concept of an 'environment' is readily acceptable; in the three-dimensional cumulonimbus model, and still more so in the model of the quasi-horizontal large-scale motion system, the concept of the 'environment' is inapplicable, but by virtue of the extended parcel theory given above, it becomes also redundant, since it is replaced by the height of the parcel, or in a virtually hydrostatic pressure distribution simply by the state of the atmosphere below the parcel, wherever it may be.

It is difficult to construct trajectories for the air in which condensation occurs. On charts for surfaces of constant dry-bulb potential temperature the flow in the area occupied by the major cloud systems seems to be pierced from below by more rapidly rising air, in which the wet-bulb potential temperature remains approximately constant. Some cloud systems are composed of a small number of distinct layers, the air within which can be traced back to particular geographical regions over which the potential temperatures in the layer of small-scale convection have distinctive characteristic values. Such horizontal inhomogeneity becomes a layering in the vertical in confluent frontal zones, and the soundings there show more or less distinctly moist layers associated with minor maxima of wind speed, whose presence is a help in following the air motion.

The isentropic relative-flow charts show that the circulations of the large-scale middle-latitude convection extend near the surface into much lower latitudes than is perhaps generally appreciated. In particular, amongst the trajectories for the flow through

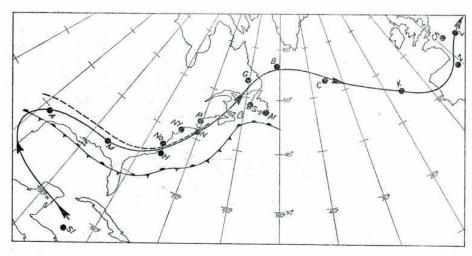


Figure 3. Representative inferred trajectory of air from the layer of small-scale convection in the trade winds which entered the Gulf of Mexico, ascended in a system of middle and high cloud over the Gulf States on 10 July 1961, and entered a jet stream over the eastern coast of the U.S.A. before traversing the Atlantic and arriving over SW Europe on 12 July. The position of the edge of the middle and high cloud system over the U.S.A. at 1200 gmr, 10 July is indicated by the pecked line, and the surface position of the quasi-stationary front at that time is shown in the conventional manner. The full circles mark the following stations, at which soundings for the indicated times are used or quoted:

			GMT	July 1961				GMT	July 1961
SI	Swan Island	78501	0000	9	Ar	Argentia	72807	0000	11
A	San Antonio	72253	1200	10	G	Goose	72816	1200	11
M	Montgomery	72226	1200	11	В	Weather ship	\boldsymbol{B}	1200	11
H	Cape Hatteras	72304	1200	10	C	Weather ship	C	1200	11
No	Norfolk	72308	1200	10	K	Weather ship	K	1200	12
NY	New York	74486	1200	10	Z	Zarogoza	08159	1200	12
N	Nantucket	72506	1200	10	L	Lyons	07480	1200	12
P	Portland	72606	1200	10	Ch	Chateauroux	07354	1200	12
S	Stephenville	72815	0000	11					

the cloud systems, those to which we have so far paid most attention leave the trade winds and ascend through the cloud system of a major cold front, to arrive in or somewhat to the right of the axis of the associated jet stream. The air which follows these trajectories has about the highest wet-bulb potential temperature of any which is involved in the large-scale convection, and it experiences the greatest acceleration. The wet-bulb potential temperature in the trade wind air is determined mainly by the sea-surface temperature, being typically about 4°C less at ship's deck level, probably 5°C less at cloud-base level (about 2,000 ft) and 8°C less at the 850 mb level. In consequence of the decrease with height, although convection from the surface ceases as the flow turns poleward into the confluence zone of the front, small-scale convection is likely to develop again when condensation occurs as a result of large-scale ascent. Evidently for this reason it is difficult to follow the early stages of this ascent, and the cloud layer produced is usually first identifiable in the middle troposphere with a wet-bulb potential temperature several degrees lower than the sea temperature along the low level part of the trajectory inferred for the air.

For example, in the trade wind flow over the Gulf of Mexico which approached a quasistationary front near the southern coast of the U.S.A. on 10 July 1961 (Fig. 3) the wet-bulb potential temperature was 24°C according to ship reports and about 20°C at the 850 mb level (Fig. 4). At 1200 GMT there was a sharply-defined edge to a system of middle and high clouds, produced over the front in the Gulf States, but whereas over southern Texas the potentially warm air is shown on soundings to extend up to about 800 mb with a θ_w of 21 to 23°C (Fig. 5), farther east the available soundings show the moist layer of the cloud system to be already above 500 mb with a θ_w of 20 to 21°C, as shown, for example, in Fig. 6. Fig. 3 includes a schematic trajectory for such air, which shows it subsequently ascending into the middle of a rather strong jet stream off the east coast of the U.S.A. It is recognizable on the sounding made at Cape Hatteras, for example (Fig. 7), by the increase in θ_w to the characteristic value of 20°C or a little more above the 370 mb level, and by the associated change of wind velocity and increase of humidity above the very dry

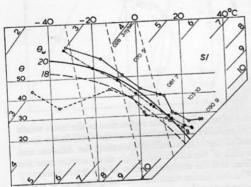


Figure 4. Sounding at Swan Island (78501; SI on Fig. 3), 0000 GMT, 9 July 1961, represented on a tephigram. In this and similar subsequent diagrams the thin horizontal lines are dry adiabatics labelled with potential temperature θ in degrees Celsius; the positions of the isobars, which slope up to the right, are indicated by short lines labelled in hundreds of mb, and curved moist adiabatics are labelled with wet-bulb potential temperatures θ_w of 16, 18 (pecked lines) and 20°C (full line). Winds are entered on the appropriate isobars, the first three digits giving the directions in degrees followed by speeds in m sec⁻¹. Temperatures and dew-points are connected by full and pecked lines respectively, and in this diagram the distribution of θ_w is shown by the dot-dash line joining the full circles; it decreases from 23°C near the surface to 20°C at the top of the moist layer of the trade winds.

Figure 5. Sounding at San Antonio (72253; A on Fig. 3), 1200 GMT, 10 July 1961. The trade wind air is thought to be present in the almost saturated layer, T, below 700 mb; notice that small-scale convection is likely to develop, as above this layer both θ_s and θ_w decrease with height. Continuous moderate rain was reported at the ground.

middle troposphere, to a state representing saturation with respect to ice. Generally humidity sensors appear incapable of responding so well, and in the analysis of the soundings it has been assumed that θ_w at high levels is given sufficiently accurately by θ_s , the saturation potential temperature. On the following day the jet stream was located near the sounding station at Goose in Canada and the Weather ships B and C (Fig. 8), while on 12 July the strong flow in the high troposphere was observed at Weather ship K and entered Spain before turning northward over France and weakening. At several sounding stations near the drawn schematic trajectory the wind speeds observed near the 300 or the 250 mb level, wherever the wet-bulb potential temperature was 20°C, are compared with the speeds calculated according to Eq. (3) and on the assumptions that the system as a whole was stationary, that over the period air left the Gulf States at the 500 mb level at 5825 m with a speed of 13 m sec⁻¹ (as indicated on the soundings in Fig. 6), and that it rose adiabatically saturated and with a wet-bulb potential temperature of 20°C. Since in the adiabatic atmosphere the thicknesses of the layer from 500 to 300 and from 500 to

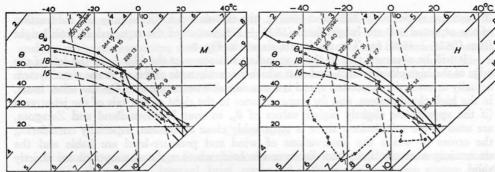


Figure 6. Sounding at Montgomery (72226; M on Fig. 3), 1200 GMT, 11 July 1961. Intermittent slight rain and an overcast of thick middle clouds were reported at the ground.

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Figure 7. Sounding at Cape Hatteras (72304; H on Fig. 3), 1200 GMT, 10 July 1961. There were no low clouds beneath an overcast of thick middle cloud.

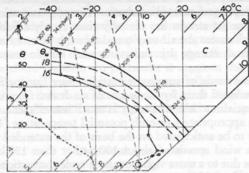


Figure 8. Sounding at Weather ship C, 1200 дмт, 11 July 1961.

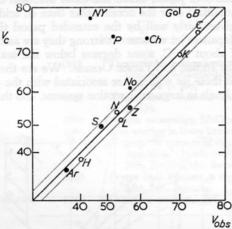


Figure 9. Comparison of wind speeds observed (Vobs) and calculated (V_c, from the extended parcel theory), in the jet stream of 10-12 July 1961. The comparison is made at the level where θ_s was observed to be 20°C, except at Argentia (Ar), Stephenville (S) and Zaragoza (Z), all on the right flank of the jet stream, where θ_s was respectively 20·7°C, 20·7°C and 21°C. The soundings at Norfolk (No), Portland (P) and New York (NY) were judged to have been made respectively 30, 70 and 100 miles to the left of the edge of the high-cloud system, and the calculated speed is increasingly an overestimate. Points are also marked for soundings at Cape Hatteras (H), Nantucket (N), Goose (G), Lyons (L), Chateauroux (Ch), and the Weather ships B, C and K. The positions of the soundings are shown on Fig. 3 and their times in its legend. Those known to have been made away from the axis of the jet are indicated by full circles. The separation of the thin diagonals corresponds to the effect of an error of plus or minus 30 m in the observed height of the 300 or 250 mb level (the ordinate and abcissa are linear in geopotential height or kinetic energy).

250 mb are respectively 3750 and 4980 m, the calculation involved only the subtraction of the reported 300 mb height from (5825 + 3750) m or of the reported 250 mb height from (5825 + 4980) m. The results are shown in Fig. 9.

It will be observed that at those sounding stations over the U.S.A. to the left of the edge of the high cloud system, and over which evidently none of the air in the cloud system actually arrived, the calculated speed is a considerable overestimate. On the other hand, in the high cloud system, and for some distance to the right of the axis of the jet-stream (if the appropriate slightly higher values of θ_s , as over Newfoundland and Zaragoza, are used in the calculation), there is remarkably close agreement, especially considering the errors to which the observations of wind and pressure-level are liable and the uncertainty over the times and distances involved, whether the assumptions are strictly valid.

Since the stratification over the Gulf States was nearly neutral for saturated adiabatic ascent, and since the surface pressure beneath the jet stream was nowhere much different from that in the Gulf States, it appears that the wind speed close to the jet stream axis may be given approximately by the area on the aerological diagram between the tropospheric sounding and the appropriate saturated adiabatic. The approximation is likely to be good only when the air in the jet stream has recently risen nearly adiabatically from the low troposphere. After such ascent the air may travel through several further motion systems, gradually losing internal energy by radiation and kinetic energy against pressure gradients.

In the few analyses made of summer situations over Europe it appears that in the high cloud systems produced there by the ascent of air drawn from over the Sahara desert the conversion of potential into kinetic energy is much less efficient. The air in the cirrus systems has about the appropriate wet-bulb potential temperature (18°C), but only about half the kinetic energy to be anticipated on the basis of the extended parcel theory, corresponding to jet stream wind speeds of about 100 rather than 150 kt. It is thought that the discrepancy may be due to a more vigorous small-scale convection during the formation of the cloud system (in the middle troposphere), but it needs further study. On the other hand, the winter jet streams over Japan, which like those over and off the eastern U.S.A. would be entered by air modified over the ocean rather than an arid continent, also have wind speeds which are given fairly well by the extended parcel theory (e.g., Fig. 10). It appears that when the Japanese jet streams are strong they are associated with wet-bulb potential temperatures of about 24°C, some degrees below the sea surface temperature in very low latitudes in the Indian and Pacific Oceans. We are therefore examining the interesting hypothesis that these jet streams are associated with the ascent of air through the troposphere from low levels in large-scale motion systems, and that the air which flows

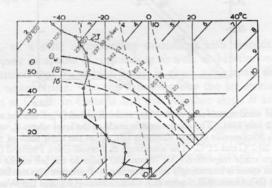


Figure 10. Temperature and wind sounding at Shionomisaki, Japan (47778), 1200 GMT, 17 January 1961. The wind speed at 280 mb according to the extended parcel theory, as given approximately by the area between the sounding and the pecked adiabatic line (for $\theta_3 = 23^{\circ}$ C), is 102 m sec⁻¹.

away from them approximately on paths of constant absolute vorticity constitutes the so-called subtropical jet streams of other longitudes. Hill (1964) who has made isentropic analyses of the corresponding subtropical winter jet streams over and off eastern Australia, has remarked upon the development of rain-bearing middle cloud systems in latitudes as low as 15 to 20°S, and has obtained a rather similar model for the sloping ascent of air from these systems into the core of the jet stream.

CONCLUSION

It is often an advantage to combine observations made at different times, on the assumption that over an interval, brief, compared with its life, a motion system under study can be regarded as in a steady state. This technique has been used in the examination, for example, of hurricanes and mesoscale weather systems, and more recently in the construction of a three-dimensional cumulonimbus model (Ludlam 1963). With the further assumption that over the same interval most of the motion is adiabatic, a technique of relative-flow analysis can be made to give some insight into large-scale baroclinic convection and a model of it which strongly resembles the cumulonimbus model. The value of the technique is partly in providing such a simple and comprehensible model and partly in raising interesting questions, for example, concerning the effect of small-scale convection within the large-scale cloud systems. In particular it suggests that in these cloud systems trade wind air ascends directly into the jet streams; it allows an extension of the simple parcel theory, and shows that the area on an aerological diagram between a saturated adiabatic and the sounding beneath a jet stream is related to its speed.

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REFERENCES

1917	'Theoretische-meteorologische Mitteilungen. 4: Die hydro- dynamische-thermodynamische Energiegleichung. Met.
1961	Zeit., 34, p. 166. 'Trajectories: isobaric, isentropic and actual,' J. Met., 18, p. 479.
1940 1964	Dynamic meteorology, McGraw-Hill, New York. 'The weather in lower latitudes of the SW Pacific associated with the passage of disturbances in the middle latitude westerlies,' Proc. Symp. Tropical Meteorology, 1963, New Zealand Met. Service, Wellington.
1963	'Severe local storms: a review,' Met. Mon., 5, 27, Amer. Met. Soc., Boston.
1937	'Isentropic analysis,' Bull. Amer. Met. Soc., 18, p. 261.
	1961 1940 1964