

On the use and significance of isentropic potential vorticity maps

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SUMMARY

The two main principles underlying the use of isentropic maps of potential vorticity to represent dynamical processes in the atmosphere are reviewed, including the extension of those principles to take the lower boundary condition into account. The first is the familiar Lagrangian conservation principle, for potential vorticity (PV) and potential temperature, which holds approximately when advective processes dominate frictional and diabatic ones. The second is the principle of 'invertibility' of the PV distribution, which holds whether or not diabatic and frictional processes are important. The invertibility principle states that if the total mass under each isentropic surface is specified, then a knowledge of the global distribution of PV on each isentropic surface and of potential temperature at the lower boundary (which within certain limitations can be considered to be part of the PV distribution) is sufficient to deduce, diagnostically, all the other dynamical fields, such as winds, temperatures, geopotential heights, static stabilities, and vertical velocities, under a suitable balance condition. The statement that vertical velocities can be deduced is related to the well-known omega equation principle, and depends on having sufficient information about diabatic and frictional processes. Quasi-geostrophic, semi-geostrophic, and 'nonlinear normal mode initialization' realizations of the balance condition are discussed. An important constraint on the mass-weighted integral of PV over a material volume and on its possible diabatic and frictional change is noted.

Some basic examples are given, both from operational weather analyses and from idealized theoretical models, to illustrate the insights that can be gained from this approach and to indicate its relation to classical synoptic and air-mass concepts. Included are discussions of (a) the structure, origin and persistence of cutoff cyclones and blocking anticyclones, (b) the physical mechanisms of Rossby wave propagation, baroclinic instability, and barotropic instability, and (c) the spatially and temporally nonuniform way in which such waves and instabilities may become strongly nonlinear, as in an occluding cyclone or in the formation of an upper air shear line. Connections with principles derived from synoptic experience are indicated, such as the 'PVA rule' concerning positive vorticity advection on upper air charts, and the role of disturbances of upper air origin, in combination with low-level warm advection, in triggering latent heat release to produce explosive cyclonic development. In all cases it is found that time sequences of isentropic potential vorticity and surface potential temperature charts—which succinctly summarize the combined effects of vorticity advection, thermal advection, and vertical motion without requiring explicit knowledge of the vertical motion field—lead to a very clear and complete picture of the dynamics. This picture is remarkably simple in many cases of real meteorological interest. It involves, in principle, no sacrifices in quantitative accuracy beyond what is inherent in the *concept* of balance, as used for instance in the initialization of numerical weather forecasts.

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APPENDIX: THE COMPUTATION OF VERTICAL MOTION

1. INTRODUCTION AND HISTORICAL REVIEW

(a) *Early ideas*

Circulation and vorticity have been recognized as fundamental concepts in meteorology and oceanography for many years, dating back to the pioneering work of V. Bjerknes (1898a, b, 1901, 1902); see also, e.g., Eliassen (1982). The three-dimensional vorticity equation, as it appears in textbooks on general fluid dynamics, may be written for frictionless motion relative to a coordinate system rotating with angular velocity Ω as

$$\frac{D}{Dt}(\zeta_a/\rho) = (\zeta_a/\rho) \cdot \nabla \mathbf{u} - (1/\rho) \nabla(1/\rho) \times \nabla p \quad (1)$$

where the absolute vorticity

$$\zeta_a = 2\Omega + \zeta \quad (2)$$

and

$$\zeta = \nabla \times \mathbf{u}, \quad (3)$$

the relative vorticity. D/Dt is the material rate of change, and ∇ is the three-dimensional gradient operator. We denote the three-dimensional velocity by $\mathbf{u} = (u, v, w)$ to distinguish it from the horizontal wind velocity $\mathbf{v} = (u, v, 0)$. The first term on the right-hand side of (1) is the stretching-twisting term, and the second the so-called solenoid term.

The absolute circulation around a material circuit Γ moving with the fluid is

$$C_a = C + 2\Omega A, \quad (4)$$

where

$$C = \oint_{\Gamma} \mathbf{u} \cdot d\mathbf{l} \quad (5)$$

is the relative circulation and A is the area bounded by a projection of the circuit onto a plane normal to $\boldsymbol{\Omega}$. Circulation is another measure of the rotational character of the air motion (which is the aspect of the air motion usually of interest in dynamical meteorology) and is equal to the integral of the vorticity over a surface bounded by the circuit. Following Bjerknes, we may write the circulation theorem for frictionless motion as

$$dC_a/dt = - \oint_{\Gamma} (1/\rho) dp. \quad (6)$$

The stretching-twisting term, the first term on the right of (1), has been absorbed into the behaviour of the material circuit and does not appear explicitly in (6).

When the usual meteorological approximations are made, i.e. neglecting vertical accelerations and the horizontal component of the rotation $\boldsymbol{\Omega}$, taking the geoid to be spherical, and replacing the distance of an air parcel from the centre of the earth by a constant representative value of the earth's radius, the equations are unaltered except that

- (i) where \mathbf{u} appears explicitly in (3) and (5) it is replaced by the horizontal wind vector \mathbf{v} ;
- (ii) only the vertical component f of $2\boldsymbol{\Omega}$ is used in (2) and (4); and
- (iii) the plane involved in the definition of the projected area A in (4) is horizontal, rather than perpendicular to $\boldsymbol{\Omega}$.

(b) *Rossby and Ertel*

In general, the complexity of the foregoing equations means that detailed argument from them is difficult. Rossby (1939) took a key step by realizing that in practice the vertical component of absolute vorticity $\zeta_a = f + \mathbf{k} \cdot (\nabla \times \mathbf{v})$, \mathbf{k} being a unit vertical vector, is the most important for the large-scale atmospheric flow. He realized furthermore that many features of the flow could be surprisingly well modelled by assuming conservation of ζ_a in two-dimensional horizontal motion—the familiar barotropic model of large-scale atmospheric dynamics. The streamfunction for this flow is obtainable at any time by inversion of the Laplacian operator linking it and the vorticity; we shall refer to this as the 'invertibility principle' for vorticity in the barotropic model. Rossby's insight, which clarified and simplified some earlier insights from what Rossby referred to as "a remarkable paper by J. Bjerknes" (1937), led to a number of important developments, for instance the theory of Rossby wave motion, the use of constant absolute vorticity trajectories, and, a decade later, the conception and execution of the first practicable numerical forecasting models.

Rossby (1940) took a further key step by noticing that if h is the depth of a material fluid column in the barotropic model, then

$$\zeta_a/h = \text{constant} \quad (7)$$

following the fluid column. This describes, in an ingeniously simple way, the two processes that often dominate the vorticity budget, namely the creation of vorticity by vertical stretching of vortex tubes, and the horizontal advection of absolute vorticity. This is the simplest version of the modern concept of 'potential vorticity'.

In the late 1930s, Rossby and his co-workers (e.g. Rossby 1937a, b; Namias

